Calibration method for a photoelastic modulator with a peak retardation of less than a half-wavelength

Aijun Zeng, Lihua Huang, Zuoren Dong, Jianming Hu, Huijie Huang, and Xiangzhao Wang

A new calibration method for a photoelastic modulator is proposed. The calibration includes a coarse calibration and a fine calibration. In the coarse calibration, the peak retardation of the photoelastic modulator is set near 1.841 rad. In the fine calibration, the value of the zeroth Bessel function is obtained. The zeroth Bessel function is approximated as a linear equation to directly calculate the peak retardation. In experiments, the usefulness of the calibration method is verified and the calibration error is less than 0.014 rad. The calibration is immune to the intensity fluctuation of the light source and independent of the circuit parameters. The method specially suits the calibration of a photoelastic modulator with a peak retardation of less than a half-wavelength. © 2007 Optical Society of America

OCIS codes: 260.5430, 230.4110.

1. Introduction

A photoelastic modulator (PEM) is a device for modulating the polarization state of light based on the photoelastic effect. The PEM is applied to many measurements of the polarization effect, the polarization transfer function, the phase map, the quantum mechanical state, the film thickness, the material characteristics, and so on.[1–6] The wide usage is due to the PEM's many superior characteristics, such as a large acceptance angle, a relatively large aperture, a large optical bandwidth, a high modulating frequency, and low power consumption.[7,8]

To ensure optimum performance, the user is required to accurately calibrate the PEM, i.e., to measure its peak retardation. A typical calibration method is the oscilloscope waveform method in which a flattop waveform is observed on the oscilloscope with half-wave peak retardation.[9,10] In many cases, the PEM can be calibrated by Bessel function zero methods.[9] For the zeroth Bessel function zero method, the direct-current term is kept invariable to set the peak retardation at 2.405 rad. For the first and second Bessel function zero methods, the fundamental and secondary harmonic components are nulled to set the peak retardation at 3.872 and 5.136 rad, respectively. For special applications, *in situ* calibration of the PEM can be realized by a single photon-counting method and a multiple-harmonic intensity ratio method.[4,11] The oscilloscope waveform method and the first and second Bessel function zero methods can accurately calibrate the PEM but are not fit for a PEM with a peak retardation of less than a half-wavelength. The zeroth Bessel function zero method is sensitive to the intensity fluctuation of the light source and relies on subjective judgement; thus the method is not used in precise calibration of the PEM. The single photon-counting method and the multiple-harmonic intensity ratio method can calibrate the PEM under any peak retardation, but these two methods suit only PEMs in the atomic collision setup with a weak signal and ellipsometry using a data acquisition system, respectively.

To accurately calibrate a PEM with a peak retardation of less than a half-wavelength, a new calibration method is proposed in this paper. First, a coarse calibration is used to set the peak retardation near 1.841 rad. Then a fine calibration is carried out to finely measure the peak retardation of the PEM.

2. Principle

The optical arrangement for the coarse calibration of the PEM is illustrated in Fig. 1. The PEM to be calibrated and a wave plate are placed between a polarizer and an analyzer. The transmission axes of the polarizer and the analyzer are perpendicular to each other. The PEM is oriented with its modulator axis at 45° with respect to the transmission axes of

The authors are with the Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China. A. Zeng's e-mail address is aijunzeng@siom.ac.cn.

Received 14 February 2006; revised 12 October 2006; accepted 18 October 2006; posted 20 October 2006 (Doc. ID 68054); published 25 January 2007.
0003-6935/07/050699-05$15.00/0

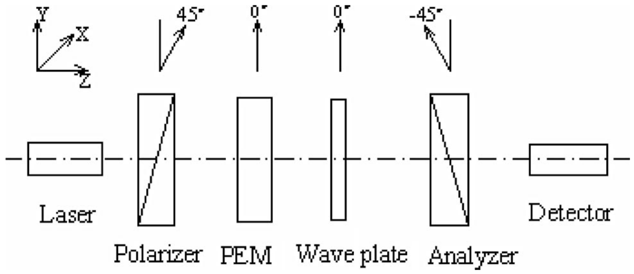

Fig. 1. Optical arrangement for the coarse calibration of the PEM.

the polarizer and the analyzer. The fast axis of the wave plate is parallel to the modulator axis of the PEM. The light source is a laser. The collimated laser beam is linearly polarized by the polarizer. The light propagates through the PEM, the wave plate, and the analyzer to form polarization interference. The interference intensity is detected by a detector.

The phase retardation $\phi(t)$ versus time t of the PEM at operating frequency ω is given by

$$\phi(t) = \alpha_0 \sin(\omega t), \tag{1}$$

where α_0 is the peak retardation of the PEM.[8,9] In the optical arrangement shown in Fig. 1, the interference intensity at the detector is given by

$$I_C = I_0[1 - \cos(\beta + \alpha_0 \sin \omega t)]/2, \tag{2}$$

where I_0 is the initial intensity subsequent to the polarizer and β is the retardation of the wave plate.[8] With Fourier series expansion, Eq. (2) can be rewritten as

$$I = I_0[1 - \cos \beta J_0(\alpha_0) + 2 \sin \beta J_1(\alpha_0) \sin \omega t - 2 \cos \beta J_2(\alpha_0) \cos 2\omega t + \cdots\cdots]/2, \tag{3}$$

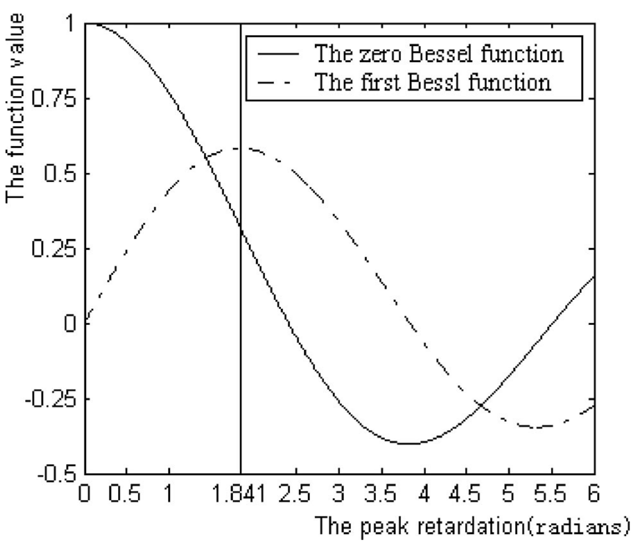

Fig. 2. Plots of Bessel functions versus the peak retardation.

where $J_0(\alpha_0)$, $J_1(\alpha_0)$, and $J_2(\alpha_0)$ are the zeroth, first, and second Bessel functions, respectively. It is obvious that the interference intensity has a complicated harmonic content related to the Bessel functions whose values are dependent on the peak retardation.

The relations of the zeroth and first Bessel functions versus the peak retardation are illustrated in Fig. 2. In the graph, a peak exists at 1.841 rad in the curve of the first Bessel function versus the peak retardation. The first-harmonic term of the interference intensity is given by

$$I_{1f} = I_0 \sin \beta J_1(\alpha_0). \tag{4}$$

The first-harmonic term reaches the maximum when the peak retardation is 1.841 rad. Thus the PEM can be calibrated by maximizing the first-harmonic term. In the calibration procedure, the peak retardation is changed until the maximum of the first-harmonic term is obtained. Thereby the peak retardation can be set near 1.841 rad.

However, the derivative of the first Bessel function is zero at the maximum, that is to say, the value of the first Bessel function is varied slowly enough when the peak retardation is changed around 1.841 rad. Thus the first-harmonic term in the neighborhood of the maximum is not sensitive enough to the variation of the peak retardation. For example, when the peak retardation is changed from 1.674 to 2.010 rad, the

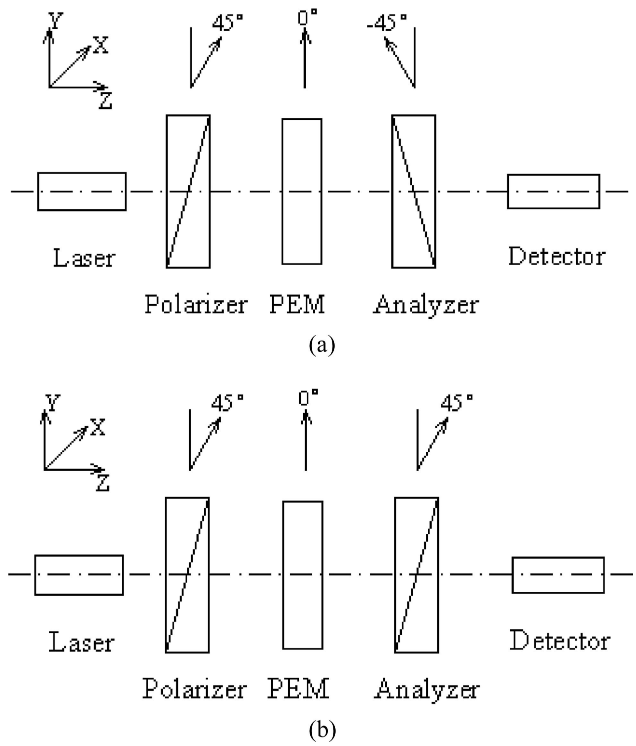

Fig. 3. Optical arrangements for the fine calibration of the PEM (a) before and (b) after the analyzer is rotated.

value of the first Bessel function is varied between 0.5761 and 0.5819. Thus it can be seen that a 9.1% change of the peak retardation around 1.841 rad results in only a 1% change of the first-harmonic term. Consequently a little measurement error of the first-harmonic term of the intensity, induced by the intensity fluctuation of the light source and the like, will bring a large calibration error. Thus the PEM is coarsely calibrated by maximizing the first-harmonic term.

The first Bessel function is the derivative of the zeroth Bessel function, namely,

$$J_0'(\alpha_0) = -J_1(\alpha_0). \tag{5}$$

When the first Bessel function reaches the maximum, the zeroth Bessel function varies most rapidly. It is obvious that the zeroth Bessel function is most sensitive to the variation of the peak retardation near 1.841 rad. Thus the peak retardation can be finely measured using the value of the zeroth Bessel function. The value of the zeroth Bessel function is obtained from the optical arrangement for the fine calibration. The optical arrangement is shown in Fig. 3. The optical arrangement shown in Fig. 3(a) is formed by taking away the wave plate from the optical arrangement shown in Fig. 1. When the analyzer shown in Fig. 3(a) is rotated by 90° to have the same orientation as the polarizer, the optical arrangement shown in Fig. 3(b) is formed.

With the coarse calibration, the actual value of the peak retardation is denoted as $\alpha_{0\,\max}$. The interference intensities in Figs. 3(a) and 3(b) are given by

$$I_{F1} = I_0[1 - J_0(\alpha_{0\,\max}) - 2J_2(\alpha_{0\,\max})\cos 2\omega t + \cdots]/2, \tag{6}$$

$$I_{F2} = I_0[1 + J_0(\alpha_{0\,\max}) + 2J_2(\alpha_{0\,\max})\cos 2\omega t + \cdots]/2, \tag{7}$$

respectively.[12] The direct-current term and the second-harmonic term (absolute value) of the interference intensity in Fig. 3(a) are expressed as

$$I_{dc1} = I_0[1 - J_0(\alpha_{0\,\max})]/2, \tag{8}$$

$$I_{2f1} = I_0 J_2(\alpha_{0\,\max}). \tag{9}$$

The direct-current term and the second-harmonic term of the interference intensity in Fig. 3(b) are expressed as

$$I_{dc2} = I_0[1 + J_0(\alpha_{0\,\max})]/2, \tag{10}$$

$$I_{2f2} = I_0 J_2(\alpha_{0\,\max}). \tag{11}$$

After the analyzer is rotated, the initial intensity is

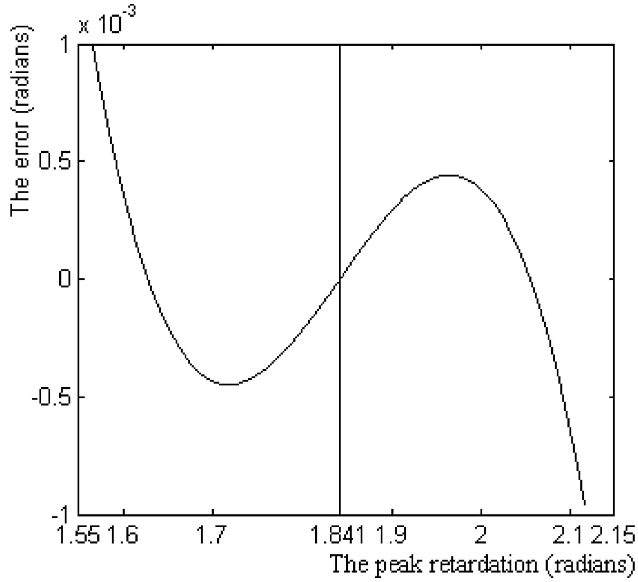

Fig. 4. Peak retardation error caused by linear approximation of the zeroth Bessel function around 1.841 rad.

possibly changed due to the intensity fluctuation of the light source. The values of the second Bessel functions in Eqs. (9) and (11) are equal. To eliminate the influence of the intensity fluctuation of the light source, the direct-current term can be normalized by the second-harmonic term. The normalized direct-current terms are written as

$$R_1 = \frac{I_{dc1}}{I_{2f1}} = \frac{1 - J_0(\alpha_{0\,\max})}{2J_2(\alpha_{0\,\max})}, \tag{12}$$

$$R_2 = \frac{I_{dc2}}{I_{2f2}} = \frac{1 + J_0(\alpha_{0\,\max})}{2J_2(\alpha_{0\,\max})}. \tag{13}$$

The two normalized direct-current terms are independent of the initial intensities. With the two normalized direct-current terms, $J_0(\alpha_{0\,\max})$ can be obtained using the sum–difference method given by

$$J_0(\alpha_{0\,\max}) = \frac{R_2 - R_1}{R_2 + R_1}. \tag{14}$$

To resolve the peak retardation from the value of the zeroth Bessel function, a general method using interpolation with a table of Bessel functions can be used. With a program such as MATLAB or Mathcad, the value of the zeroth Bessel function can be calculated for a given peak retardation. When the peak retardation is varied with a small step, a series of values of the zeroth Bessel function are obtained to form a table of the zeroth Bessel function. With this table, the peak retardation can be accurately determined using interpolation.

From Fig. 2, it can be seen that the value of the zeroth Bessel function varies approximately linearly

Table 1. Results of the Coarse Calibration

α_0 (rad)	1.750	1.765	1.780	1.795	1.810	1.825	1.840	1.855	1.870	1.885	1.900	1.915	1.930
I_{1f} (V)	7.351	7.368	7.385	7.389	7.394	7.381	7.386	7.384	7.383	7.380	7.373	7.365	7.356

with the peak retardation near 1.841 rad. That is to say, the curve of the zeroth Bessel function versus the peak retardation can be considered as a line when the α_0 is near 1.841 rad. Thus the peak retardation can be easily and directly resolved from a linear function that is obtained by fitting the curve of the zeroth Bessel function as a line with the least-squares method. For example, when the peak retardation is in the range of 1.565 to 2.117 rad, where the peak retardation varies by 15% and the value of the first Bessel function is changed by 2.7%, the zeroth Bessel function can be expressed approximately as

$$J_0(\alpha_0) = -0.5787\alpha_0 + 1.3816. \tag{15}$$

With the approximate equation, the peak retardation can be easily calculated with

$$\alpha_{0\,\max} = [1.3816 - J_0(\alpha_{0\,\max})]/0.5787. \tag{16}$$

When the peak retardation is in the range of 1.565–2.117 rad, the peak retardation error generated by the calculation with Eq. (16) is expressed as

$$\alpha_{\text{error}} = [1.3816 - J_0(\alpha_0)]/0.5787 - \alpha_0. \tag{17}$$

With a MATLAB program, the peak retardation error α_{error} is calculated and given in Fig. 4. In Fig. 4, the largest error is less than 0.001 rad in the retardation range between 1.6 and 2.1 rad. It is seen that the peak retardation error generated by the calculation with Eq. (16) is very small and its influence on the calibration can be ignored. Thus the peak retardation can be calculated from the value of the zeroth Bessel function with high precision using Eq. (16).

With the optical arrangement for the coarse calibration, the peak retardation is set near 1.841 rad by maximizing the first-harmonic term. The PEM is calibrated at a peak retardation of less than a half-wavelength. With the optical arrangement for the fine calibration, the direct-current terms normalized by the second-harmonic terms are obtained to resolve the value of the zeroth Bessel function using the sum–difference method given by Eq. (14). Then the peak retardation is directly calculated using the approximate Eq. (16). In the fine calibration, the normalization of the direct-current terms can eliminate the influence of the intensity fluctuation that exists in the coarse calibration. When the value of the zeroth Bessel function is obtained using the sum–difference method, the influence of the circuit parameters relative to the direct-current terms and the second-harmonic terms in the signal process can be removed. Thus the fine calibration is independent of the parameter of the signal processing circuit. From the value of the zeroth Bessel function, the peak retardation is resolved by approximating the zeroth Bessel function as the linear equation. Accordingly, it is made direct and easy to calculate the peak retardation with high precision.

3. Experiment

In our experiment, the light source was a laser diode with an aspheric collimating lens. The laser wavelength was 785 nm. The spectral width (FWHM) of the laser was 1 nm. The polarizer and the analyzer were Glan–Taylor prisms with an extinction ratio greater than 100000:1. The PEM was a Hinds model I/FS50 PEM with a 50 kHz modulation frequency. The static retardation of the PEM was about 0.5 nm. The maximum error of the zeroth Bessel function in Eq. (14) induced by the static retardation was about 0.0008% with the analytical method in Ref. 8. The error can be neglected. The optical element of the PEM was 0.25 in. thick (1 in. = 2.54 cm). The thickness was much greater than the coherent length of the laser. The modulated interference effect may be neglected. Thus the experiment on the PEM was valid. The peak retardation of the PEM was controlled by a PEM-90 controller. Before the experiment, the PEM had been calibrated by the first Bessel function zero method. The peak retardation of the PEM was used as the reference in the experiment. The wave plate is an achromatic quarter-wave plate over the spectral interval from 700 to 1100 nm. The detector was a photodiode. In the electronic system for signal processing, a Hinds SCU-90 signal conditioning unit, a lock-in amplifier, and a data acquisition device were used.

The polarizer, the PEM, the wave plate and the analyzer were adjusted to form the optical arrangement for the coarse calibration shown in Fig. 1 with the light extinction method. The controller was adjusted to change the peak retardation of the PEM. When the peak retardation α_0 varied from 1.760 to 1.910 rad with a step of 0.015 rad, the first-harmonic

Table 2. Results of the Fine Calibrations

α_0 (rad)	1.750	1.765	1.780	1.795	1.810	1.825	1.840	1.855	1.870	1.885	1.900	1.915	1.930
$\alpha_{0\,\mathrm{M}}$ (rad)	1.739	1.757	1.778	1.784	1.801	1.820	1.830	1.849	1.860	1.873	1.892	1.901	1.923
Calibration error (rad)	0.011	0.008	0.002	0.011	0.009	0.005	0.010	0.006	0.010	0.012	0.008	0.014	0.007

terms were obtained. A typical result of the coarse calibration is presented in Table 1. These values fluctuate only in the range of 7.370–7.393 V. It is obvious that the first-harmonic term is not sensitive enough to the variation of the peak retardation. The maximum of the first-harmonic term occurs when the peak retardation is 1.810 rad. Thereby the precision of the coarse calibration is not high.

Then the wave plate was removed to form the optical arrangement for the fine calibration shown in Fig. 3. Before and after the analyzer was rotated, the peak retardation α_0 varied from 1.76 to 1.910 rad with a step of 0.015 rad. The direct-current terms and the second-harmonic terms at each measuring point were obtained to resolve the peak retardation using Eq. (16). At all measuring points, the results of the fine calibrations are given in Table 2. $\alpha_{0\,M}$ is the measured value of the peak retardation. The calibration error is the difference between the measured value $\alpha_{0\,M}$ and the actual value α_0. From Table 2, it can be seen that the calibration error is less than 0.014 rad. Thus the usefulness of the calibration method is verified by the experiment.

4. Conclusion

A new calibration method for a PEM has been proposed in this paper. The calibration is immune to the intensity fluctuation of the light source and is independent of the parameters of the signal processing circuit. The method specially suits the calibration of the PEM whose peak retardation is less than a half-wavelength. In the experiments, the usefulness of the calibration method is verified and the calibration error is less than 0.014 rad.

This work was supported by the National Natural Science foundation of China under grant 60578051.

References

1. M. Kuldkepp, N. C. Hawkes, E. Rachlew, and B. Schunke, “Accurate polarization measurements with a dual photoelastic modulator,” Appl. Opt. **44,** 5899–5904 (2005).
2. M. Mujat and A. Dogariu, “Real-time measurement of the polarization transfer function,” Appl. Opt. **40,** 34–44 (2001).
3. A. Dubois, “Phase-map measurements by interferometry with sinusoidal phase modulation and four integrating buckets,” J. Opt. Soc. Am. A **18,** 1972–1979 (2001).
4. R. A. Cline, W. B. Westerveld, and J. S. Risley, “A new method for measuring the retardation of a photoelastic modulator using single photon counting techniques,” Rev. Sci. Instrum. **64,** 1169–1174 (1993).
5. H. L. Maynard, N. Layadi, and J. T. C. Lee, “Plasma etching of submicron devices: *in situ* monitoring and control by multiwavelength ellipsometry,” Thin Solid Films **313–314,** 398–405 (1998).
6. Y. Shindo, “Application of polarized modulation technique in polymer science,” Opt. Eng. **34,** 3369–3384 (1995).
7. J. Badoz, M. P. Silverman, and J. C. Canit, “Wave propagation through a medium with static and dynamic birefringence: theory of the photoelastic modulator,” J. Opt. Soc. Am. A **7,** 672–682 (1990).
8. J. C. Cheng, L. A. Nafie, S. D. Allen, and A. I. Braunstein, “Photoelastic modulator for the 0.55–13 μm range,” Appl. Opt. **15,** 1960–1965 (1995).
9. *PEM-90 Photoelastic Modulator Systems User Manual* (Hinds Instruments Inc., Hillsboro, Oreg., 1998).
10. T. C. Oakberg, J. Trunk, and J. C. Sutherland, “Calibration of photoelastic modulators in the vacuum UV,” in *Polarization Analysis, Measurement, and Remote Sensing III,* D. B. Chenault, M. J. Duggin, W. G. Egan, and D. H. Goldstein, eds., Proc. SPIE **4133,** 101–112 (2000).
11. M. W. Wang, F. H. Tsai, and Y. F. Chao, “*In-situ* calibration technique for photoelastic modulator in ellipsometry,” Thin Solid Films **455–456,** 78–83 (2004).
12. J. C. Canit and J. Badoz, “New design for a photoelastic modulator,” Appl. Opt. **22,** 592–594 (1983).